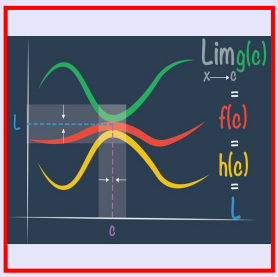
Math 261

Spring 2023

Lecture 56



Feb 19-8:47 AM

Class QZ 18

Evaluate $\displaystyle\int_0^R x\sqrt{R^2-x^2}\,dx$, $R \geq 0$

$u = R^2 - x^2$

$du = -2x\,dx$

$\dfrac{du}{-2} = x\,dx$

Exact answer only.

All work must be clearly displayed.

$x=0 \to u=R^2$

$x=R \to u=0$

$$\int_0^R x\sqrt{R^2-x^2}\,dx = \int_{R^2}^{0} \sqrt{u}\cdot\frac{du}{-2} = \frac{-1}{2}\int_{R^2}^{0} u^{1/2}\,du$$

$$= \frac{-1}{2}\cdot -\int_0^{R^2} u^{1/2}\,du = \frac{1}{2}\cdot\frac{u^{3/2}}{3/2}\Big]_0^{R^2} = \frac{1}{3}u\sqrt{u}\Big]_0^{R^2}$$

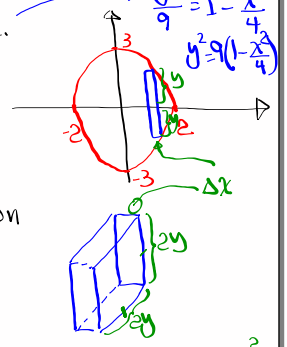
$$= \frac{1}{3}\left[R^2\sqrt{R^2} - 0\sqrt{0}\right] = \boxed{\frac{R^3}{3}}$$

May 24-9:32 AM

The base of the Solid S is an elliptical shape with equation $\frac{x^2}{4}+\frac{y^2}{9}=1$.

$\frac{y^2}{9}=1-\frac{x^2}{4}$

$y^2=9\left(1-\frac{x^2}{4}\right)$



Cross-Sections ⊥ x-axis.

Cross-Sections are Squares.

one Side of Square is on the base of Solid S.

Find its Volume.

$A(x)=2y\cdot 2y=4y^2$

$$V=\int_{-2}^{2}A(x)\,dx=\int_{-2}^{2}4y^2\,dx$$

$$=4\int_{-2}^{2}9\left(1-\frac{x^2}{4}\right)dx=\int_{-2}^{2}\underbrace{(36-9x^2)}_{\text{even function}}dx$$

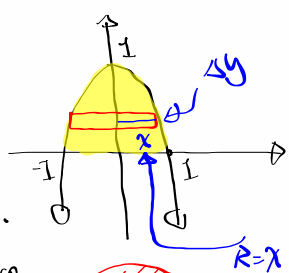
$$=2\int_{0}^{2}(36-9x^2)\,dx=2\left(36x-\frac{9x^3}{3}\right)\Big]_0^2$$

$$=2\left(36x-3x^3\right)\Big]_0^2=\boxed{}$$

May 25-8:50 AM

The base of solid S is the region enclosed by $y=1-x^2$ and x-axis.

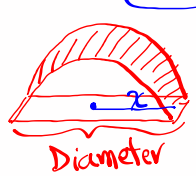
$x^2=1-y$



Cross-Sections ⊥ Y-axis.

Cross-Section are Semi-Circle.

Diameter of Cross-Section are in the enclosed region.



Find its Volume.

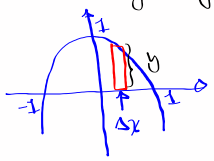
$A(x)=\frac{\pi R^2}{2}=\frac{\pi x^2}{2}$

$$V=\int_0^1 A(x)\,dy=\int_0^1\frac{\pi x^2}{2}\,dy=\frac{\pi}{2}\int_0^1(1-y)\,dy$$

$$=\frac{\pi}{2}\left(y-\frac{y^2}{2}\right)\Big]_0^1=\frac{\pi}{2}\cdot\frac{1}{2}=\boxed{\frac{\pi}{4}}$$

May 25-8:59 AM

The base of Solid S is enclosed region by $y=1-x^2$ and x-axis.



Cross-Sections $\perp$ x-axis.

Cross-Sections are isosceles triangles with height equal to base.

Base of the Cross-Sections is on the enclosed region.

Find its Volume.

$$A(x)=\frac{bh}{2}=\frac{y\cdot y}{2}=\frac{y^2}{2}$$

$$V=\int_{-1}^{1}\frac{y^2}{2}dx=\frac{1}{2}\int_{-1}^{1}y^2dx$$

$$=\frac{1}{2}\int_{-1}^{1}\underbrace{(1-x^2)^2}_{\text{Even}}dx=\frac{1}{2}\cdot 2\int_{0}^{1}(1-x^2)^2dx$$

$$=\int_0^1(1-2x^2+x^4)\,dx=\left(x-\frac{2x^3}{3}+\frac{x^5}{5}\right)\Big]_0^1$$

$$=1-\frac{2}{3}+\frac{1}{5}=\frac{1}{3}+\frac{1}{5}=\boxed{\frac{8}{15}}$$

May 25-9:06 AM

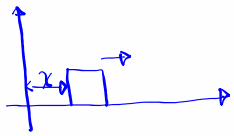
Last Topic:

Work

If we apply a force given by $f(x)$ to an object to move it from a to b, the required work is

$$W=\int_a^b f(x)dx$$

ex: An object is x units from the origin and a force of x^2+4x pounds acts on it. How much work is required to move it from 1 ft to 4 ft.



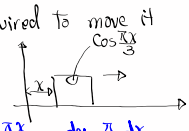
$$W=\int_1^4(x^2+4x)dx$$

$$=\left(\frac{x^3}{3}+2x^2\right)\Big]_1^4=\boxed{}$$

ft-lb.

May 25-9:15 AM

A particle is located x ft from the origin. A force of $\cos\left(\frac{\pi x}{3}\right)$ pounds acts on it. How much work is required to move it from 1 ft to 2 ft.



$u = \frac{\pi x}{3}$ $\quad du = \frac{\pi}{3}dx$

$\frac{3}{\pi}du = dx$

$$W = \int_1^2 \cos\frac{\pi x}{3}\,dx$$

$x=1 \to u = \frac{\pi}{3}$

$x=2 \to u = \frac{2\pi}{3}$

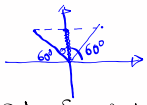
$\pi/3 = 60^\circ$, $2\pi/3 = 120^\circ$

$$= \int_{\pi/3}^{2\pi/3} \cos u \cdot \frac{3}{\pi}\,du$$

$$= \frac{3}{\pi}\left(\sin u\right)\Big]_{\pi/3}^{2\pi/3} = \frac{3}{\pi}\left[\sin\frac{2\pi}{3} - \sin\frac{\pi}{3}\right]$$

$$= \frac{3}{\pi}\left[\underbrace{\sin 120^\circ - \sin 60^\circ}_{0}\right]$$

$$= \boxed{0}$$



Redo for $x=1$ to $x=1.5$

$x=1 \quad u = \pi/3$

$x=1.5 \quad u = \frac{\pi}{3}\cdot\frac{3}{2} = \frac{\pi}{2}$

$$= \frac{3}{\pi}\int_{\pi/3}^{\pi/2} \cos u\,du = \frac{3}{\pi}\left[\sin u\right]_{\pi/3}^{\pi/2}$$

$$= \frac{3}{\pi}\left[\sin\pi/2 - \sin\pi/3\right] = \frac{3}{\pi}\left[1 - \frac{\sqrt{3}}{2}\right]$$

Redo for $x=1.5$ to $x=2$

$$= \frac{3}{\pi}\int_{\pi/2}^{2\pi/3} \cos u\,du = \frac{3}{\pi}\left(\sin u\right)\Big]_{\pi/2}^{2\pi/3} = \frac{3}{\pi}\left(\sin\frac{2\pi}{3} - \sin\frac{\pi}{2}\right)$$

$$= \frac{3}{\pi}\left[\frac{\sqrt{3}}{2} - 1\right]$$

May 25-9:22 AM

Class QZ 19

A particle is x units from the origin. A force of $\frac{5}{x^2}$ acts on it. Find the work required to move the particle from $x=1$ to $x=10$. Exact answer only.

$$W = \int_1^{10} \frac{5}{x^2}\,dx$$

$$= 5\int_1^{10} x^{-2}\,dx$$

$$= 5\cdot\frac{x^{-1}}{-1}\Big]_1^{10}$$

$$= -5\left(\frac{1}{x}\right)\Big]_1^{10} = -5\left(\frac{1}{10} - \frac{1}{1}\right) = -5\cdot\frac{-9}{10} = \boxed{\frac{9}{2}} = \boxed{4.5}$$

May 25-9:36 AM